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A DECOMPOSITION-BASED DESIGN OPTIMIZATION METHOD WITH APPLICATIONS

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TOPICS

There are many real-world engineering design problems which cannot be effectively handled using conventional design optimization methods. Special techniques and/or modifications of the conventional methods are necessary to handle such complex problems. One solution involves two-level decomposition, whereby a problem is divided into smaller subproblems, each with its own design objective and constraints (refs. 1,2).

Here, we will describe a two-level design optimization methodology and give a progress report of its application to Printed Wiring Board (PWB) assembly examples.

1. **Two-Level Design Optimization**
   - **Formulation**
   - **Procedure**
2. **Example: PWB Assembly**
3. **Summary**

FORMULATION

We consider a problem which may be decomposed into two-levels, each having several local variables. Here, i,j are the indices corresponding to the number of subproblems and number of constraints in each subproblem, respectively. Furthermore, x_i is the vector of "local" design variables in the lower-level subproblem i, and y is the vector of "global" design variables in the top-level problem.

$$\text{Minimize } f(y;x) = f_o(y) + \sum_{i=1}^{I} f_i(y;x_i)$$

$$\text{Subject: } g_\ell(y) < 0 \qquad \ell = 1,\ldots,L$$

$$g_{i,j}(y;x_i) < 0 \qquad j = 1,\ldots,J$$

PROCEDURE

The procedure is to

(1) Select the starting value for the global variables y,
(2) find x_i (y is fixed), i=1,...,I, in subproblem i,
(3) find a new y in the top-level problem such that f(y,x) is decreased,
(4) return to step (2) until the minimum for f(y;x) is obtained.

Subproblem i:

Minimize $f_i(y;x_i)$

Subject to: $g_{i,j}(y;x_i)<0 \qquad j=1,\ldots,J$

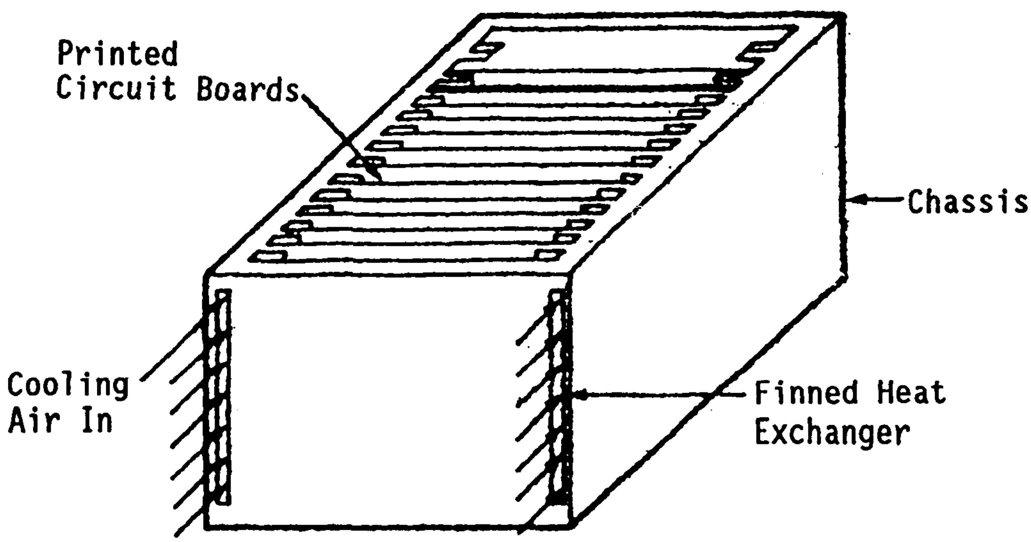
Top-level problem:

$$\text{Minimize } f(y;x) = f_o(y) + \sum_{i=1}^{I} f_i(y;x_i)$$

Subject to: $g_\ell(y)<0 \qquad \ell=1,\ldots,L$

EXAMPLE: PWB ASSEMBLY

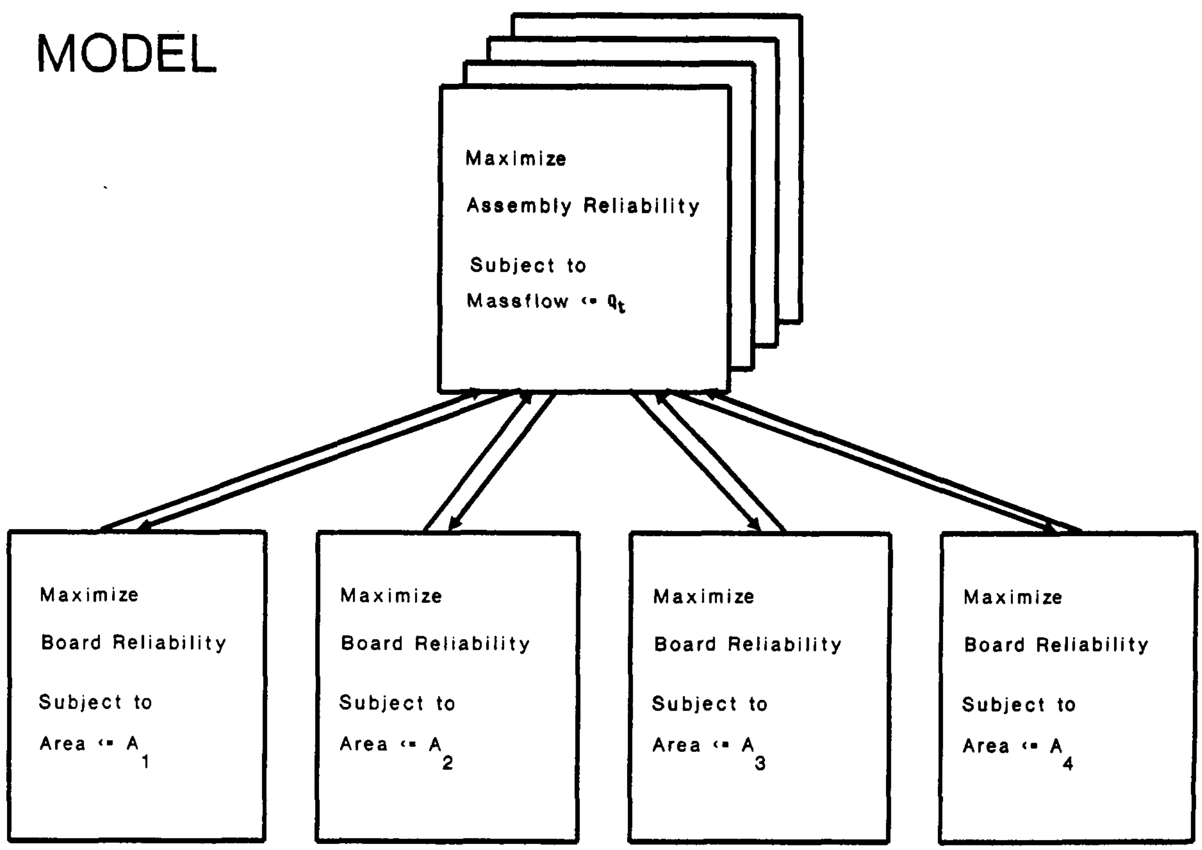
Design optimization of a PWB assembly is considered. The objective is to determine the required component redundancy and fluid flow-rate for each PWB such that the reliability of the assembly is maximized. This is a mixed-integer nonlinear programming problem.



EXAMPLE: TWO-LEVEL MODEL OF A PWB ASSEMBLY

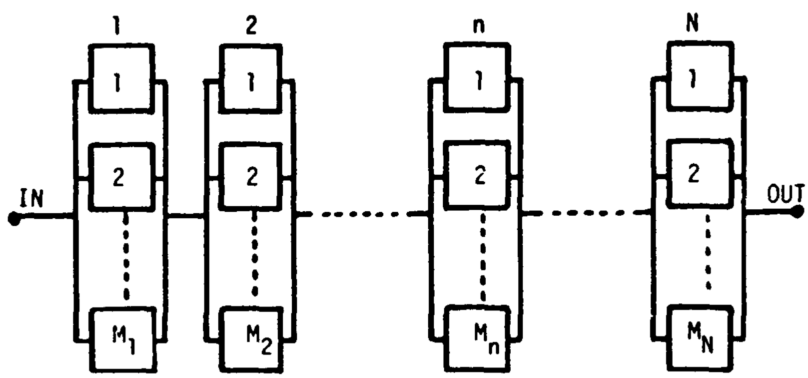
Here, a two-level design optimization model for an assembly of PWBs is presented. Allocation of fluid flow-rates (continuous variables) is performed at the top-level problem, while, allocation of component redundancy (integer variables) for each PWB is performed at the bottom-level subproblems.

MODEL



EXAMPLE: REDUNDANCY ALLOCATION FOR A PWB (Ref. 3)

It is assumed that each PWB consists of a series of N stages, where each stage n, is a parallel combination of M_n redundant components. All components in a stage are active. Thus, for a stage to fail, all components in that stage must fail. Furthermore, it is assumed that for a PWB, all components at a given stage are identical and equally reliable.



EXAMPLE: TWO-LEVEL OPTIMIZATION FORMULATION

In the two-level formulation of the PWB assembly, subproblem i corresponds to PWB i in which the reliability (R_i) is maximized. In the top-level problem, fluid flow-rates (Q_i, i=1,...,I) are allocated to maximize the assembly reliability (R). It is assumed that the assembly is a series system of I PWBs.

Subproblem i:

$$\text{Maximize } R_i = \prod_{n=1}^{N} (1-q_n^{M_n})$$

$$\text{Subject to: } \sum_{n=1}^{N} A_n M_n e^{M_n \beta} - A_{avi} < 0$$

$$M_n > 1 \qquad n=1,\ldots,N$$

Top-Level Problem:

$$\text{Maximize } R = \prod_{i=1}^{I} R_i$$

$$\text{Subject to: } \sum_{i=1}^{I} Q_i - Q_t < 0$$

$$Q_i > 0 \qquad i=1,\ldots,I$$

where:

q_n = nth stage component unreliability of ith PWB
M_n = nth stage component redundancy of ith PWB
A_{avi} = available area of ith PWB
A_n = area of a component at nth stage of ith PWB
Q_t = total fluid flow-rate of assembly

EXAMPLES

Three PWB assembly examples were solved. The first example was an assembly of 2 PWBs, each PWB having 5 stages. The second example was an assembly of 4 PWBs, each PWB having 15 stages. The third example was an assembly of 4 PWBs, each PWB having 30 stages. The overall design objective in each example was to maximize assembly reliability.

Example	No. of stages/PWB	No. of PWBs	Variables	Constraints
1	5	2	12	15
2	15	4	64	69
3	30	4	124	129

EXAMPLE: RESULTS

Initial and final solutions for an assembly of two PWBs are given:

INITIAL:
5 STAGES PER PWB

ASSEMBLY RELIABILITY = 0.908061

Q_t = 2.0 lbs/min

PWB1 Reliability = 0.953124

Q_1 = 0.5 lbs./min

M_n = (1,1,1,1,1)

PWB2 Reliability = 0.95272

Q_2 = 0.5 lbs./min

M_n = (1,1,1,1,1)

FINAL:
5 STAGES PER PWB

ASSEMBLY RELIABILITY = 0.99778

Q_t = 2.0 lbs./min

PWB1 Reliability = 0.997828

Q_1 = 1.34 lbs./min

M_n = (3,2,1,1,2)

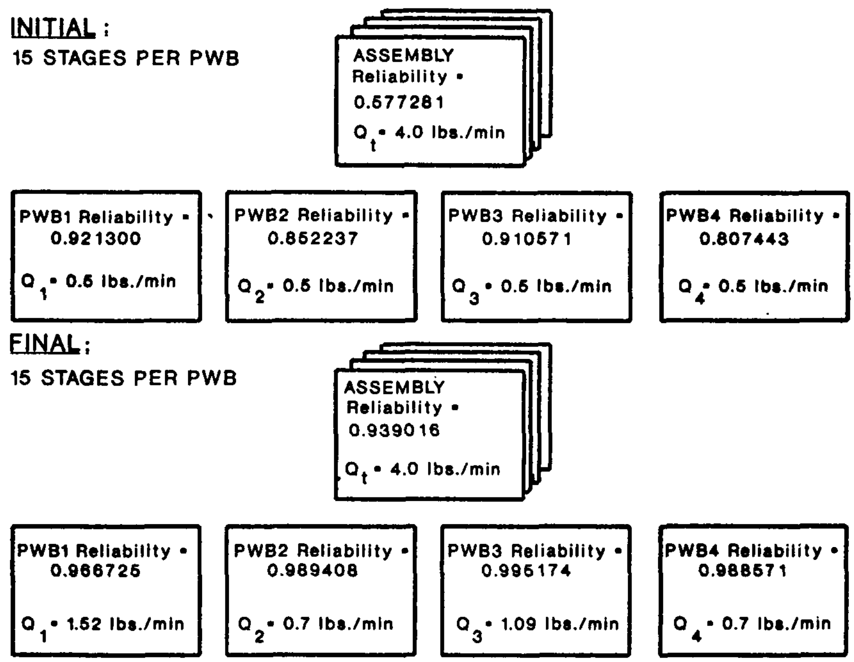
PWB2 Reliability = 0.999953

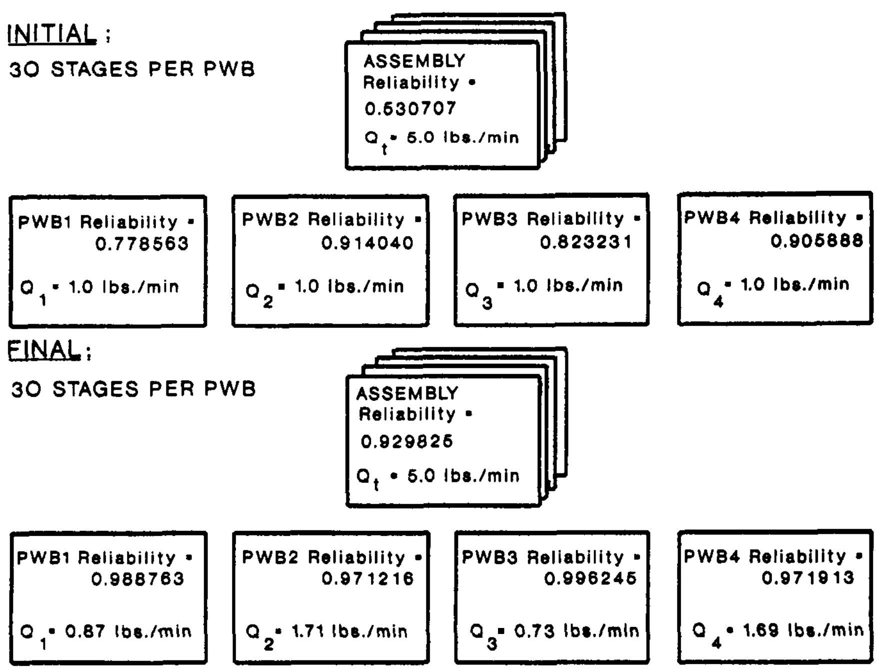
Q_2 = 0.66 lbs./min

M_n = (3,4,2,2,2)

EXAMPLE: RESULTS

Initial and final solutions for two assemblies of four PWBs are given.





SUMMARY

The design of PWB assemblies is a complex task which is generally conducted as a "sequential process." Individual PWBs are usually designed first, followed by the composition of the PWBs into an assembly. As a result, optimizing design considerations such as assembly reliability cannot be accomplished. This study showed that a two-level decomposition method can be employed to optimize for reliability at both the PWB- and the assembly-level in a coupled manner. The two-level decomposition method also resolved the mixed-integer nonlinear programming nature of the problem rather easily.

- **The sequential design process makes system optimization impossible**
- **A mixed-integer nonlinear optimization problem modelled and solved using a two-level optimization technique**
- **More research is needed to improve the performance of the two-level optimization method**

REFERENCES

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